INFINITE–DIMENSIONAL APPROACH TO SYSTEM IDENTIFICATION OF SPACE CONTROL LABORATORY EXPERIMENT (SCOLE)

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ABSTRACT

The identification of a unique set of system parameters in large space structures poses a significant new problem in control technology. This paper presents an infinite–dimensional identification scheme to determine system parameters in large flexible structures in space. The method retains the distributed nature of the structure throughout the development of the algorithm and a finite–element approximation is used only to implement the algorithm. This approach eliminates many problems associated with model truncation used in other methods of identification. The identification problem is formulated in Hilbert space and an optimal control technique is used to minimize weighted least squares of error between the actual and the model data. A variational approach is used to solve the problem. A costate equation, gradients of parameter variations and conditions for optimal estimates are obtained. Computer simulation studies are conducted using a shuttle–attached antenna configuration, more popularly known as the Space Control Laboratory Experiment (SCOLE) as an example. Numerical results show a close match between the estimated and true values of the parameters.

DISTRIBUTED PARAMETER IDENTIFICATION

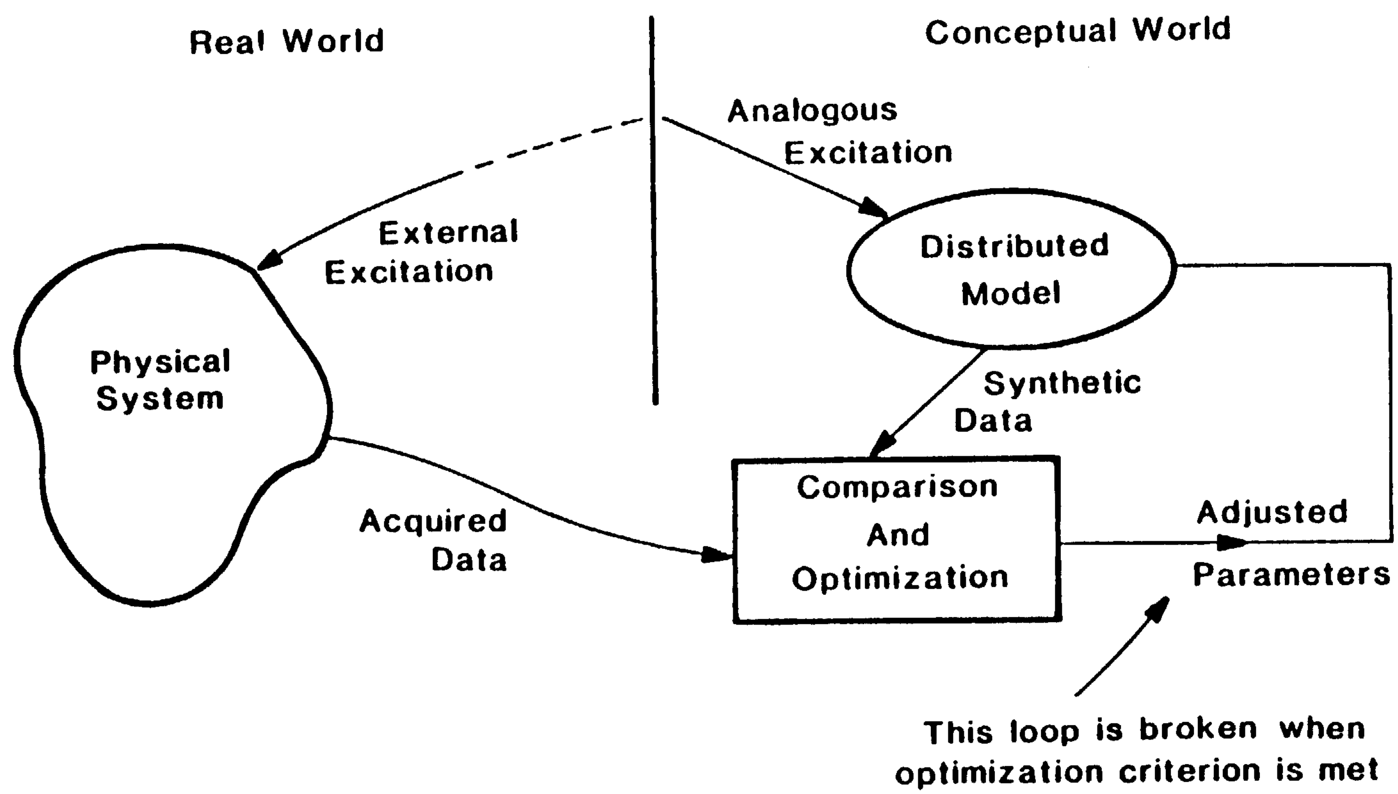
TWO APPROACHES :

FINITE-DIMESIONAL METHOD
INFINITE-DIMENSIONAL METHOD

Table 1

Literature Surveyed on the Parameter Estimation of Large Space Structures.

Reference	Approach
Wells and Spalding (1977) [2] Tung (1981) [3] Balas and Lilly (1981) [4] Balas (1981) [5] Lee and Bitter (1981) [6] Banks (1982) [7] Hendricks et al (1982) [8] Hendricks et al (1984) [9] Banks and Rosen (1984) [10] Rajaram and Junkins (1985) [11] Lee, Walker and Hossain (1985) [12] Lee (1986) [13]	A finite-dimensional design approach where the structural model is truncated and the estimator is designed based on the reduced-order model.
Spalding (1976) [14] Burns and Cliff (1977)[15] Sun and Juang (1982) [16] Lee (1986) [13]	An infinite-dimensional design approach where the PDE model is retained as long as possible and truncation is carried out only after the estimation algorithm is developed.



SYSTEM IDENTIFICATION SCHEME

A Distributed System Model

$$m(x)\frac{\partial^2 u}{\partial t^2}(x,t) + D_0\frac{\partial u}{\partial t}(x,t) + A_0 u(x,t) = F(x,t),$$

$$x \in \Omega, \qquad t \in (0,T],$$

$$F(x,t) = F_B(x,t) + F_C(x,t) + F_D(x,t),$$

$$F_C(x,t) = B_0 f = \sum_{i=1}^{M} b_i(x) f_i(t),$$

$$F_B(x,t) = B_b g = \sum_{i=1}^{N} b_i(x) g_i(t),$$

$$y = C_0 u + E_0 u_t,$$

$$J(q) = \frac{1}{2T}\int_0^T (y - z)^T R(t)(y - z)\, dt,$$

where z is the measurement of output vector y given as

$$z(x,t) = y(x,t) + e(x,t)$$

with a measurement error $e(x,t)$. Also, it is defined that

$$(y - z)^T R(t)(y - z) = \int_\Omega [y(x,t) - z(x,t)]^T R(x,t)[y(x,t) - z(x,t)]\, dx,$$

Infinite-Dimensional Formulation

$$\frac{\partial^2}{\partial t^2}u(t) + D(q)\frac{\partial}{\partial t}u(t) + A(q)u(t) = B(q)f(t) \qquad \text{in}(0,T],$$

$$u \in L_2(0,T;V), \qquad \frac{\partial u}{\partial t} \in L_2(0,T;H),$$

where $f(t)$ is given in $L_2(0,T;V)$, and the initial conditions are

$$u(0) = u_0, \qquad u_0 \text{ given in } V,$$

and

$$\frac{\partial}{\partial t}u(0) = u_1, \qquad u_1 \text{ given in } H.$$

The output function is

$$y(t) = Cu(t),$$

The identification problem can now be formulated as an abstract problem of determining the parameter vector $q^*(x) \in Q$ that minimizes

$$J(q) = \frac{1}{2T}\int_0^T [y(t) - z(t)]^T R(t)[y(t) - z(t)]\,dt,$$

where $z(t)$ is the observed data belonging to Y

THEOREM : Given a state equation (18) with initial conditions given by Eq. (19) and the cost function by Eq. (22) with y(t) satisfying Eq. (21), then the optimal parameter vector q^* satisfies the state equations (18)-(19) and the follwing system of equations :

$$\frac{d^2}{dt^2}p(t) - D^*\frac{d}{dt}p(t) + A^*p(t) = -\frac{1}{T}C^T R(Cu - z), \tag{23}$$

with the final conditions

$$p(T) = \frac{d}{dt}p(T) = 0, \tag{24}$$

and the first variation of an augmented cost functional is

$$\delta J_a = \int_0^T p^T \frac{\partial}{\partial q}\big[D\frac{du}{dt} + Au - Bf\big]\, \delta q\, dt = 0, \tag{25}$$

where $p(t)$ is a costate variable also belonging to the Hilbert space V.

PROOF : By combining Eqs. (18) and (22) an augmented cost functional can be defined as

$$\begin{aligned} J_a(q) = &\frac{1}{2T}\int_0^T [y(t) - z(t)]^T R(t)[y(t) - z(t)]\, dt \\ &+ \int_0^T p(t)^T\big[\frac{d^2}{dt^2}u(t) + D\frac{du}{dt} + Au(t) - Bf(t)\big]\, dt. \end{aligned} \tag{26}$$

Parameter Identification of Vibrating Beams

Case I : A Simply-Supported Beam

$$\rho A\frac{\partial^2 u}{\partial t^2}+EI\frac{\partial^4 u}{\partial x^4}=b(x)f(t),\qquad x\in[0,L],\, t>0,$$

$$u(x,t)=\frac{\partial^2}{\partial x^2}u(x,t)=0,\qquad x\in\partial[0,L],\quad t>0,$$

$$u(x,0)=\frac{\partial}{\partial t}u(x,t)\Big|_{t=0}=0,\qquad x\in[0,L].$$

$$y(t)=u(\frac{L}{2},t).$$

$$z(t)=y(t)+e(t).$$

$$J=\frac{1}{2T}\int_0^T[y-z]^T R[y-z]\,dt.$$

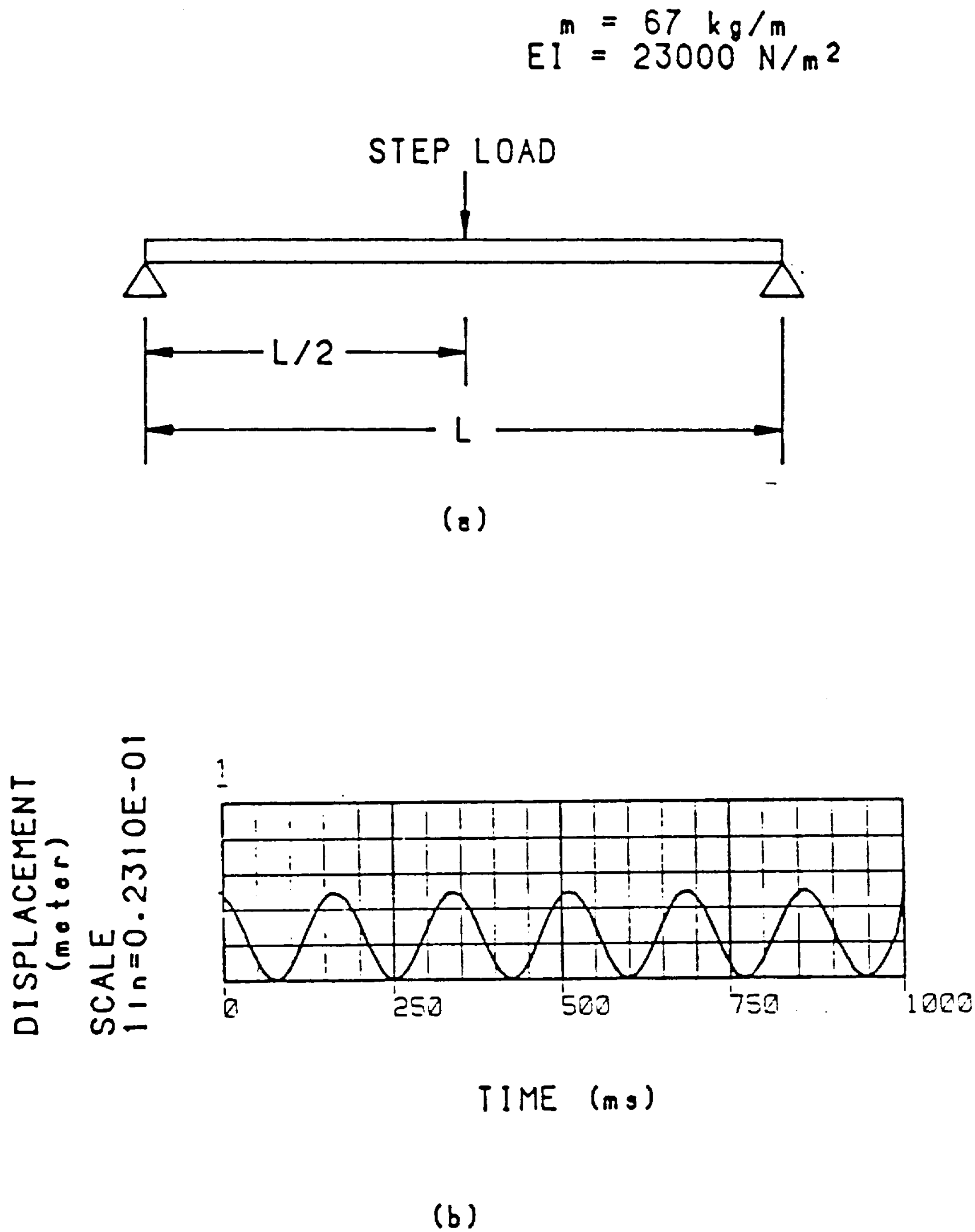


Fig. 2 (a) Simply-supprted beam with step load,
(b) resultant displacements.

$$\frac{\partial^2 u}{\partial t^2} = -\frac{EI}{m}\frac{\partial^4 u}{\partial x^4} + \frac{1}{m}\delta(x - \frac{L}{2})f(t),$$

or

$$\frac{\partial^2 u}{\partial t^2} = -q_1\frac{\partial^4 u}{\partial x^4} + q_2\delta(x - \frac{L}{2})f(t),$$

where

$$q_1 = \frac{EI}{m},$$

$$q_2 = \frac{1.}{m},$$

and the parameter vector is defined by $q = [q_1, q_2]^T$.

$$\frac{\partial^2 p}{\partial t^2} = -q_1\frac{\partial^4 p}{\partial x^4} + \frac{R}{T}[u - z]\delta(x - \frac{L}{2}), \qquad x \in [0, L], t \in [0, T),$$

$$p(x, T) = \frac{\partial}{\partial t}p(x, t)\Big|_{t=T} = 0, \qquad x \in [0, L].$$

$$p(x, t) = \frac{\partial^2}{\partial x^2}p(x, t) = 0, \qquad x \in \partial[0, L], t \in [0, T).$$

$$\frac{\delta J_a}{\delta q_1} = \int_0^T \frac{\partial^2 p}{\partial x^2}\frac{\partial^2 u}{\partial x^2}\,dt,$$

and

$$\frac{\delta J_a}{\delta q_2} = -\int_0^T p\,\delta(x - \frac{L}{2})f(t)\,dt.$$

Thus, parameters can be updated by the steepest descent algorithm

$$q_i^{k+1} = q_i^k - w_i\left(\frac{\delta J_a}{\delta q_i}\right)^k, \qquad i = 1, 2.$$

Table 2

Performance Data for Case I

Iteration	q_1	q_2	m	EI	$\int_0^T \text{error}^2\, dt$
1	301.58	0.0158	63.00	19000.00	0.20178E-01
2	311.68	0.0154	64.84	20207.72	0.16640E-01
3	325.65	0.0151	66.27	21582.87	0.71528E-02
4	338.48	0.0149	66.91	22647.68	0.57208E-03
5	342.11	0.0149	66.98	22913.64	0.34507E-04
6	342.96	0.0149	66.99	22973.13	0.26524E-04
7	343.19	0.0149	66.99	22989.35	0.24364E-04
True values	343.28	0.0149	67.00	23000.00	

Case II : A Cantileverd Beam

$$m\frac{\partial^2 u}{\partial t^2} - 2\xi\sqrt{mEI}\frac{\partial^3 u}{\partial x^2 \partial t} + EI\frac{\partial^4 u}{\partial x^4} = b(x)f(t),$$
$$x \in [0, L],\ t > 0,$$

where ξ is a damping coefficient.

$$y(t) = u(L, t).$$

The boundary conditions for a cantilevered beam are :

$$u(0,t) = \left.\frac{\partial^2}{\partial x^2}u(x,t)\right|_{x=0} = 0, \qquad t > 0,$$
$$\left.\frac{\partial^2}{\partial x^2}u(x,t)\right|_{x=L} = \left.\frac{\partial^3}{\partial x^3}u(x,t)\right|_{x=L} = 0, \qquad t > 0.$$

The beam is initially at rest and hence the initial conditions are

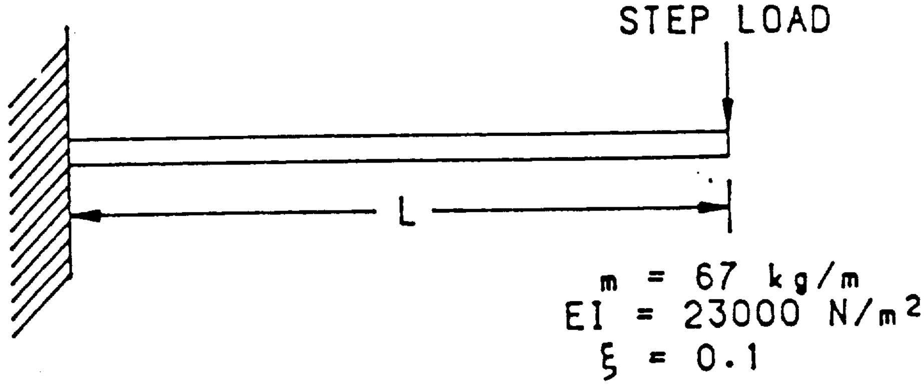
$$u(x,0) = \left.\frac{\partial}{\partial t}u(x,t)\right|_{t=0} = 0, \qquad x \in [0, L].$$
$$\left.\frac{\partial u(x,t)}{\partial x}\right|_{t=0} = \left.\frac{\partial^2}{\partial x^2}u(x,t)\right|_{t=0} = 0, \qquad x \in [0, L].$$

$$\frac{\partial^2 u}{\partial t^2} = q_3\frac{\partial^3 u}{\partial x^2 \partial t} - q_1\frac{\partial^4 u}{\partial x^4} + q_2\delta(x - L)f(t),$$

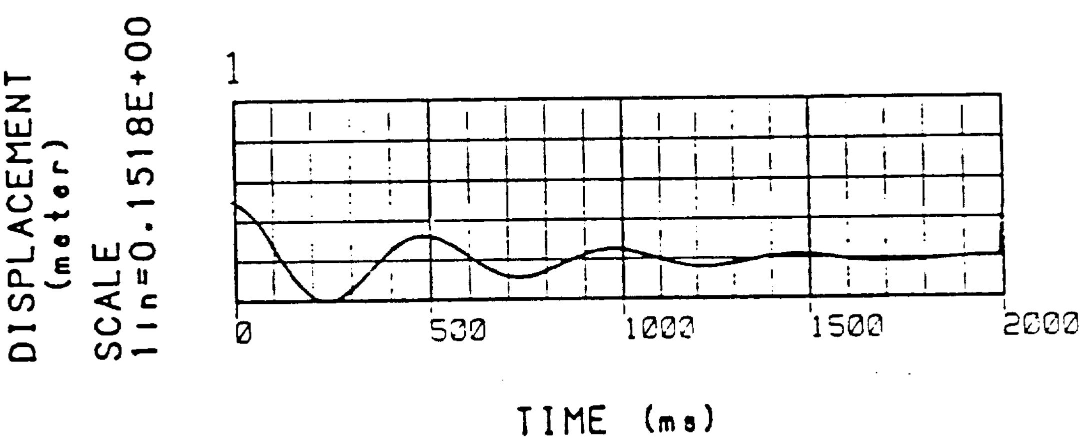
where

$$q_1 = \frac{EI}{m},$$
$$q_2 = \frac{1}{m},$$
$$q_3 = 2\xi\sqrt{\frac{EI}{m}},$$

and the parameter vector is defined by $q = [q_1, q_2, q_3]^T$



(a)



(b)

Fig. 3 (a) Cantilevered beam with a step load,
(b) resultant displacements.

Table 3

Performance Data for Case II

Iteration	q_1	q_2	q_3	m	EI	ξ	$\int_0^T \text{error}^2\, dt$
1	301.59	0.0159	2.08	63.00	19000.00	0.060	0.7843E-01
2	318.01	0.0155	2.76	64.50	20512.92	0.077	0.1664E-01
3	326.20	0.0153	3.06	65.26	21286.34	0.085	0.9432E-02
4	331.30	0.0152	3.23	65.73	21775.47	0.089	0.4498E-02
5	334.72	0.0151	3.34	66.05	22107.25	0.091	0.2284E-02
6	337.10	0.0151	3.43	66.27	22339.85	0.093	0.1238E-02
7	338.83	0.0151	3.49	66.43	22510.09	0.095	0.6649E-03
8	340.07	0.0150	3.53	66.55	22632.61	0.096	0.3796E-03
9	340.96	0.0150	3.55	66.65	22721.78	0.096	0.2199E-03
10	341.56	0.0150	3.56	66.71	22783.93	0.096	0.1519E-03
11	342.06	0.0150	3.59	66.76	22835.96	0.097	0.8667E-04
12	342.44	0.01497	3.62	66.80	22874.19	0.098	0.4961E-04
13	342.73	0.01496	3.64	66.83	22903.95	0.098	0.1165E-04
True values	343.28	0.0149	3.70	67.00	23000.00	0.1	

Case III : A Simply-Supported Beam with Spatially Variable Parameter

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(q(x)\frac{\partial^2 u}{\partial x^2}\right) = b(x)f(t), \qquad x \in [0, L],\ t > 0,$$

where $q = EI(x)$.

$$\frac{\partial^2 p}{\partial t^2} = -\frac{\partial^2}{\partial x^2}\left(q(x)\frac{\partial^2 p}{\partial x^2}\right) - \frac{R}{T}[u - z], \qquad x \in [0, L],\ t \in [0, T),$$

$$\frac{\delta J_a(x)}{\delta q} = \int_0^T \frac{\partial^2 p}{\partial x^2}\frac{\partial^2 u}{\partial x^2}\, dt.$$

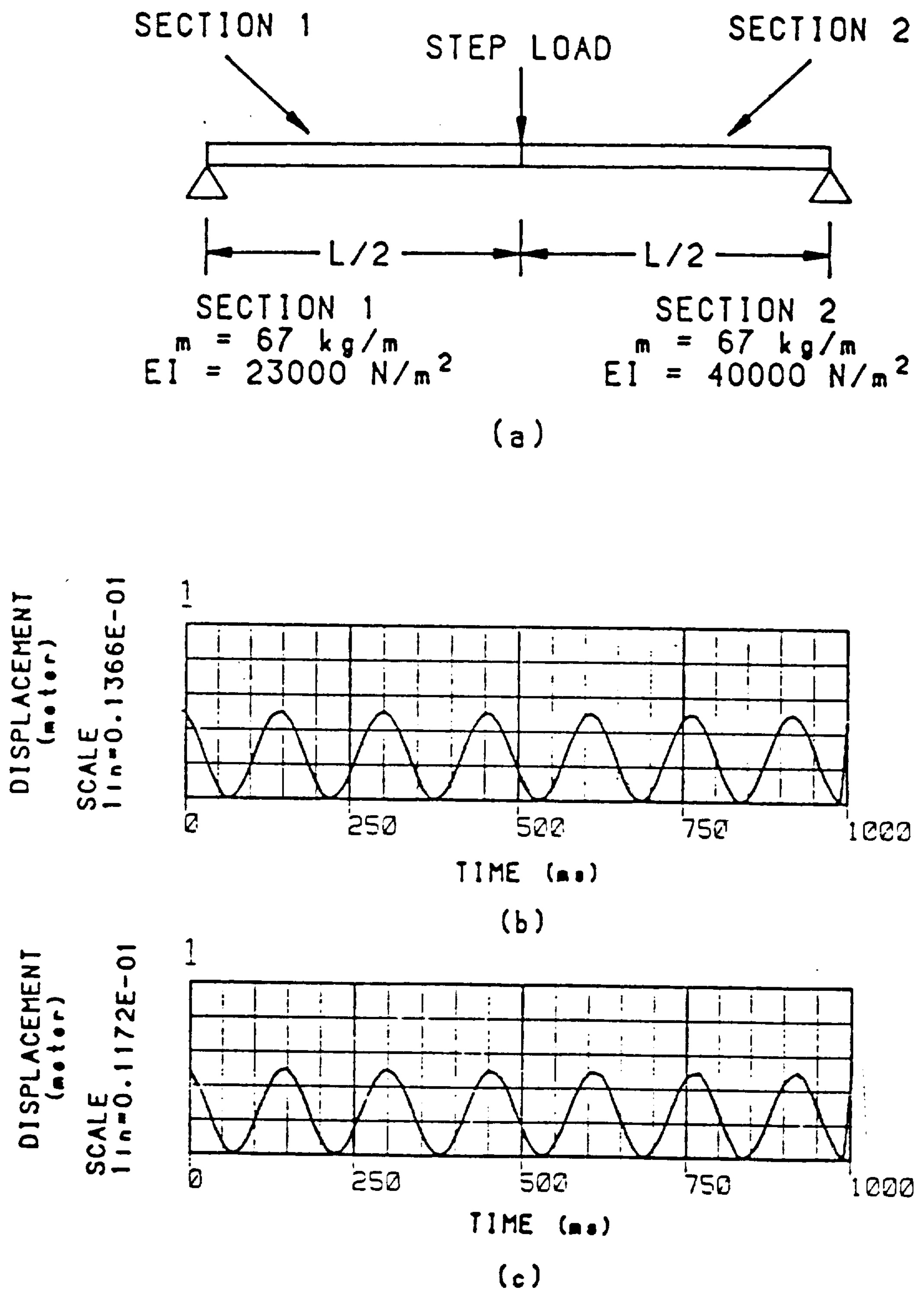


Fig. 4 (a) Simply-supported beam with spatially variable flexible rigidity,
(b) resultant displacements at L/4,
(c) resultant displcements at 3L/4.

Table 4

Performance Data for Case III

Iteration	q(sec. 1)	q(sec. 2)	$\int_0^T \text{error}^2\, dt$
1	21000.00	39000.00	0.76863E-02
2	21686.15	39286.47	0.40601E-02
3	22261.30	39521.45	0.14636E-02
4	22630.66	39688.41	0.41442E-03
5	22814.49	39759.51	0.13629E-03
6	22911.04	39793.95	0.51151E-04
7	22968.10	39815.07	0.11209E-04
True values	23000.00	40000.00	

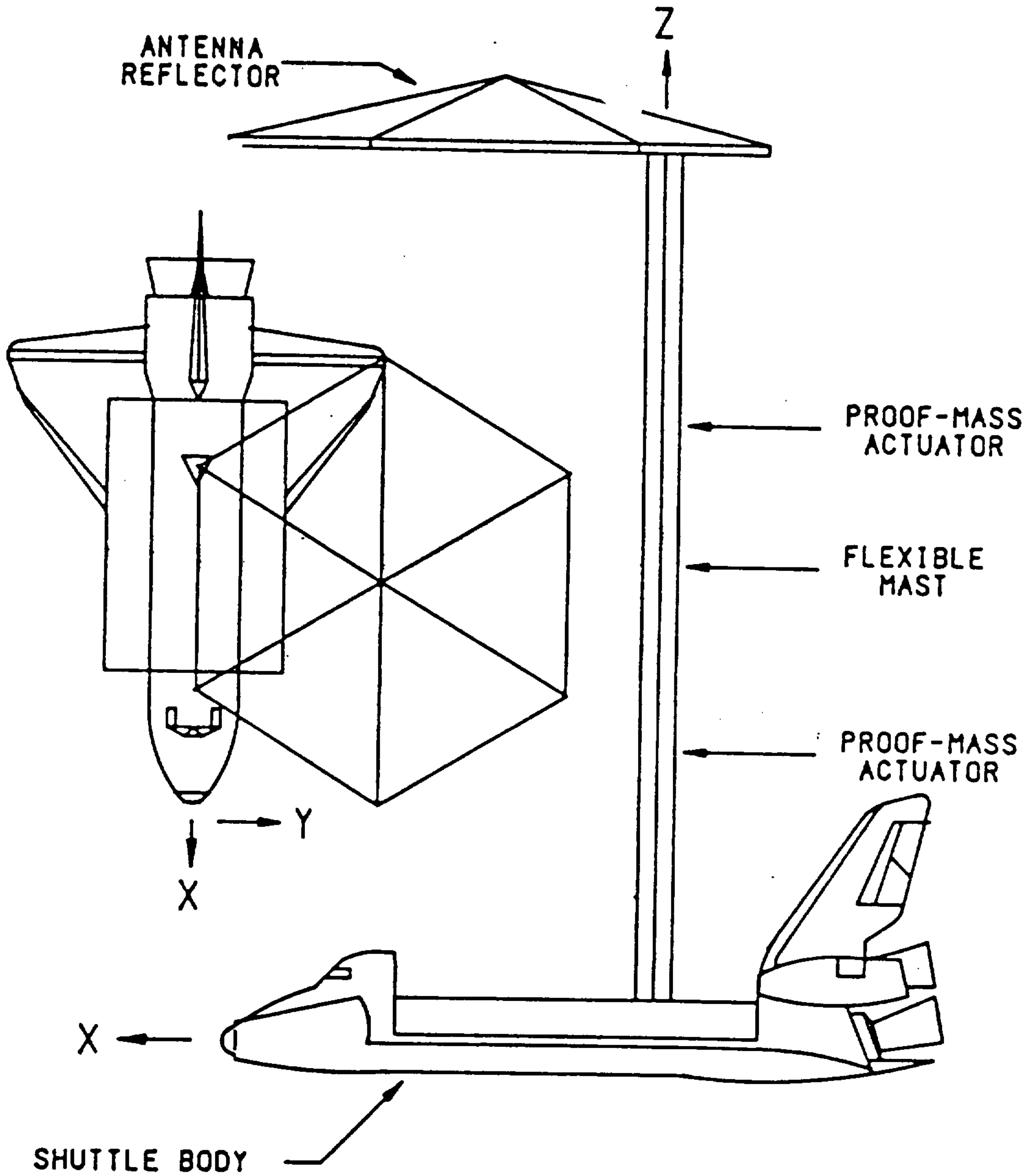


Fig. 1 The shuttle / antenna configuration of the spacecraft control laboratory experiment (SCOLE).

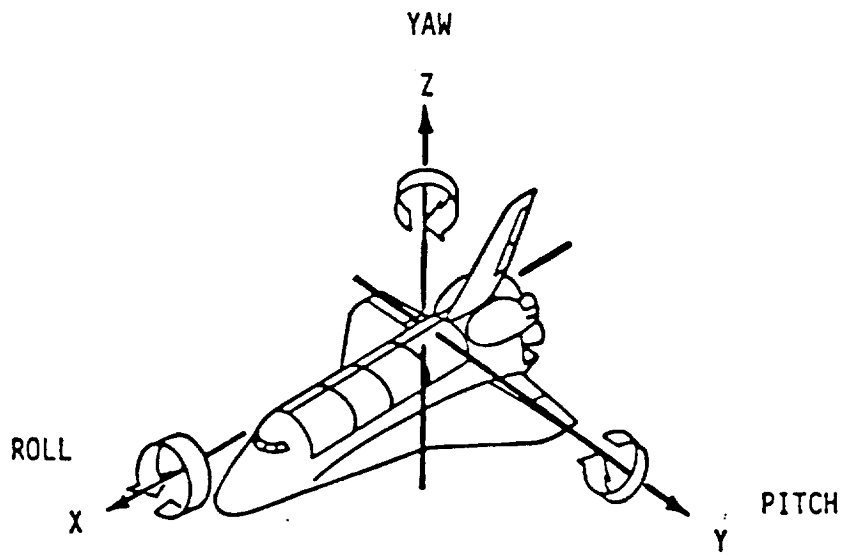


Fig. 2 Drawing showing the direction of "roll" bending, "pitch" bending and "yaw" twisting.

The SCOLE Mathematical Model

A. Dynamic Equations

Roll Beam Bending Equation in y-z Plane

$$\rho A \frac{\partial^2 u_\phi}{\partial t^2} - 2\xi_\phi \sqrt{\rho A E I_\phi} \frac{\partial^3 u_\phi}{\partial s^2 \partial t} + E I_\phi \frac{\partial^4 u_\phi}{\partial s^4}$$

$$= \sum_{n=1}^{4} \left(f_{\phi,n}(t)\delta(s - s_n) + g_{\phi,n}(t) \frac{\partial}{\partial s}\delta(s - s_n) \right),$$

$$t \geq 0, \quad -\infty < s < \infty, \quad 0 \leq s_n \leq L.$$

Pitch Beam Bending Equation in x-z Plane

$$\rho A \frac{\partial^2 u_\theta}{\partial t^2} - 2\xi_\theta \sqrt{\rho A E I_\theta} \frac{\partial^3 u_\theta}{\partial s^2 \partial t} + E I_\theta \frac{\partial^4 u_\theta}{\partial s^4}$$

$$= \sum_{n=1}^{4} \left(f_{\theta,n}(t)\delta(s - s_n) + g_{\theta,n}(t) \frac{\partial}{\partial s}\delta(s - s_n) \right),$$

$$t \geq 0, \quad -\infty < s < \infty, \quad 0 \leq s_n \leq L.$$

Yaw Beam Torsion Equation for z-Axis

$$\rho I_\psi \frac{\partial^2 u_\psi}{\partial t^2} - 2\xi_\psi I_\psi \sqrt{G\rho} \frac{\partial^3 u_\psi}{\partial s^2 \partial t} - G I_\psi \frac{\partial^2 u_\psi}{\partial s^2}$$

$$= \sum_{n=1}^{4} g_{\psi,n}(t)\delta(s - s_n),$$

$$t \geq 0, \quad -\infty < s < \infty, \quad 0 \leq s_n \leq L.$$

B. Forcing Functions

The forcing functions on the right side of each equation are dependent on boundary conditions and proof-mass actuators.

Forces at $s = s_1 = 0$ (shuttle body forces)

The forces at $s_1 = 0$ involves the shears at that point which are equal to the shuttle mass m_1 times the corresponding component of acceleration.

$$f_{\phi,1}(t) = -m_1 \frac{\partial^2}{\partial t^2} u_\phi(0,t),$$
$$f_{\theta,1}(t) = -m_1 \frac{\partial^2}{\partial t^2} u_\theta(0,t).$$

Forces at $s = s_4 = L$ (reflector body forces)

$$f_{\phi,4}(t) = -m_4 \frac{\partial^2}{\partial t^2} u_\phi(L,t) - m_4 r_x \frac{\partial^2}{\partial t^2} u_\psi(L,t) - F_y,$$
$$f_{\theta,4}(t) = -m_4 \frac{\partial^2}{\partial t^2} u_\theta(L,t) + m_4 r_y \frac{\partial^2}{\partial t^2} u_\psi(L,t) + F_x,$$

where m_4 is the reflector mass, (r_x, r_y) is center of reflector mass from the beam tip at $s = L$, and F_x and F_y are the applied forces at the center of the reflector mass.

Forces at $s = s_2$ (proof-mass actuator forces)

$$f_{\phi,2}(t) = -m_2 \frac{\partial^2}{\partial t^2} u_\phi(s_2,t) + m_2 \frac{\partial^2}{\partial t^2} \Delta_{\phi,2},$$
$$f_{\theta,2}(t) = -m_2 \frac{\partial^2}{\partial t^2} u_\theta(s_2,t) + m_2 \frac{\partial^2}{\partial t^2} \Delta_{\theta,2},$$

where Δ and m denote displacement and mass of the proof-mass actuator.

Forces at $s = s_3$ (proof-mass actuator force)

$$f_{\phi,3}(t) = -m_3 \frac{\partial^2}{\partial t^2} u_\phi(s_3,t) + m_3 \frac{\partial^2}{\partial t^2} \Delta_{\phi 3},$$
$$f_{\theta,3}(t) = -m_3 \frac{\partial^2}{\partial t^2} u_\theta(s_3,t) + m_3 \frac{\partial^2}{\partial t^2} \Delta_{\theta,3}.$$

C. Moments

<u>Moments at $s = 0$ (shuttle body moments)</u>

$$\begin{pmatrix} g_{\phi,1} \\ g_{\theta,1} \\ g_{\psi,1} \end{pmatrix} = -[I_1 \dot{w}_1 + w_1 \otimes I_1 w_1] + M_1(t) + M_D(t),$$

where I_1 is the moment of inertia of the shuttle body, $M_1(t)$ and $M_D(t)$ are control and disturbance moments, respectively, applied to the shuttle body, and $\otimes$ denotes the vector product.

<u>Moments at $s = L$ (reflector body moments)</u>

$$\begin{pmatrix} g_{\phi,4} \\ g_{\theta,4} \\ g_{\psi,4} \end{pmatrix} = -\left(\hat{I}_4 \dot{w}_4 + w_4 \otimes \hat{I}_4 w_4 - M_4(t) + r \otimes F_4(t)\right) - m_4 r \otimes \frac{\partial^2 \xi_4}{\partial t^2},$$

where M_4 and F_4 are the control moment and force applied at the reflector center of the mass and ξ_4 is the coordinates of the beam tip.

Also, I_4 is the moment of inertia of the reflector, and $\hat{I}_4$ is that with respect to the beam tip given by

$$\hat{I}_4 = I_4 + m_4 \begin{pmatrix} r_y^2 & -r_x r_y & 0 \\ -r_x r_y & r_x^2 & 0 \\ 0 & 0 & r_x^2 + r_y^2 \end{pmatrix}$$

$$M_0\ddot{r}(t) + A_0 r(t) + B_0 F(t) + K_0(\dot{r}(t)^2) = 0 \ ,$$

where M_0 is the 17×17 matrix specified by

$$\begin{array}{cc}
 & \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} \end{matrix} \\
\begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \\ x_{16} \\ x_{17} \end{matrix} &
\begin{pmatrix}
\rho A & 0 & 0 & & & & & & & & & & & & & & \\
0 & \rho A & 0 & & & & & & & & & & & & & & \\
0 & 0 & \rho A & & & & & & & & & & & & & & \\
 & & & m_1 & 0 & & & & & & & & & & & & \\
 & & & 0 & m_1 & & & & & & & & & & & & \\
 & & & & & m_4 & 0 & 0 & 0 & 0 & 0 & 0 & m_4 r_x & & & & \\
 & & & & & 0 & m_4 & 0 & 0 & 0 & 0 & 0 & -m_4 r_y & & & & \\
 & & & & & 0 & 0 & & & & 0 & 0 & 0 & & & & \\
 & & & & & 0 & 0 & & I_1 & & 0 & 0 & 0 & & & & \\
 & & & & & 0 & 0 & & & & 0 & 0 & 0 & & & & \\
 & & & & & 0 & 0 & 0 & 0 & 0 & & & & & & & \\
 & & & & & 0 & 0 & 0 & 0 & 0 & & I_4 & & & & & \\
 & & & & & m_4 r_x & -m_4 r_y & 0 & 0 & 0 & & & & & & & \\
 & & & & & & & & & & & & & m_2 & 0 & & \\
 & & & & & & & & & & & & & 0 & m_2 & & \\
 & & & & & & & & & & & & & & & m_3 & 0 \\
 & & & & & & & & & & & & & & & 0 & m_3
\end{pmatrix}
\end{array}$$

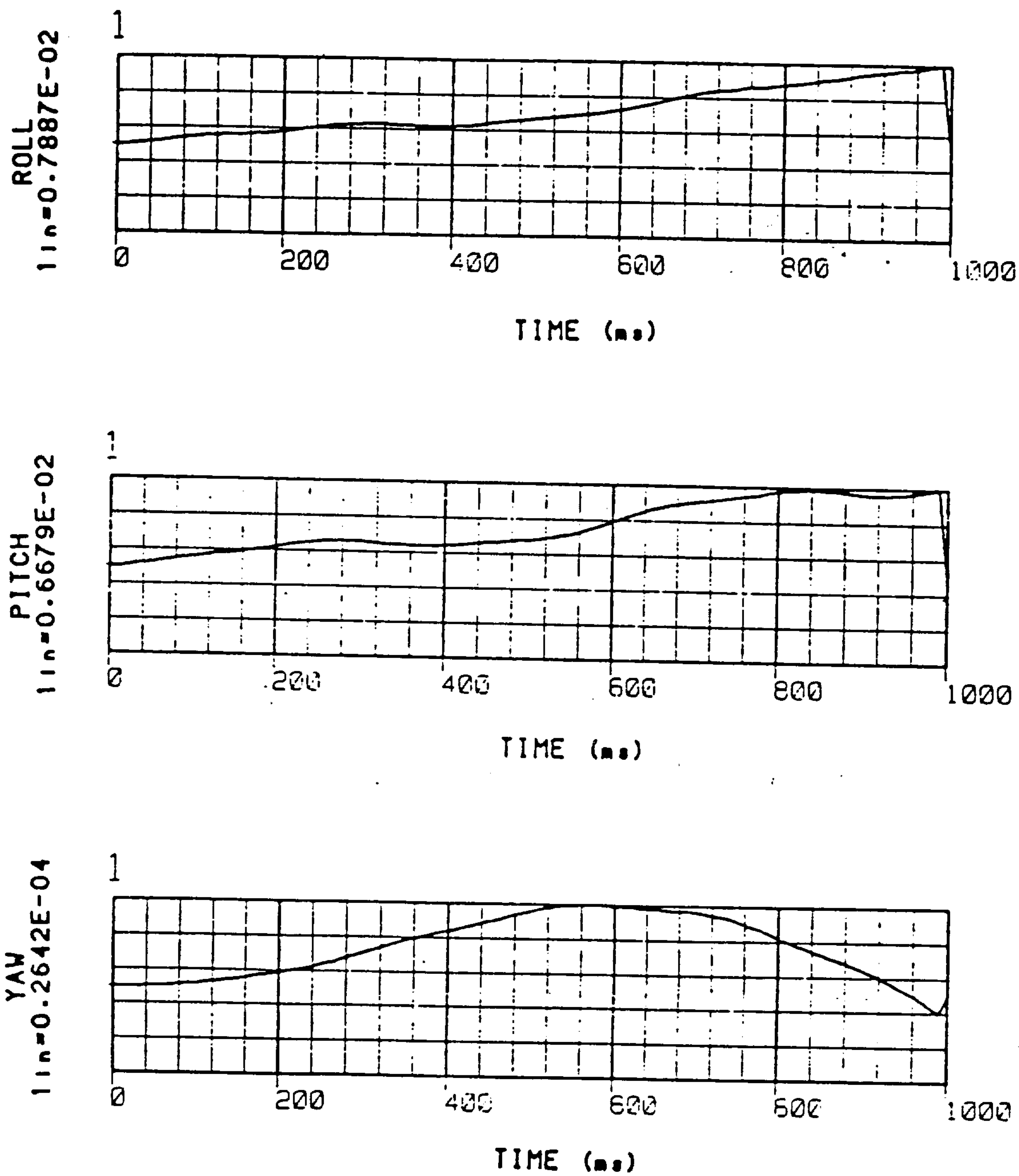


Fig. 3 Roll, pitch and yaw displacements with no damping.

Table 2

Performance Data for Case I : Nonlinear SCOLE Model

Iteration	q_1	q_2	m	EI	$\int_0^T \text{error}^2\, dt$
1	4.2353E+08	11.765	0.0850	3.6000E+07	0.1341E-04
2	4.2531E+08	11.067	0.0904	3.8427E+07	0.1896E-05
3	4.2612E+08	10.809	0.0925	3.9422E+07	0.3038E-06
4	4.2743E+08	10.705	0.0934	3.9929E+07	0.9131E-07
5	4.2704E+08	10.665	0.0938	4.0041E+07	0.8031E-07
6	4.2617E+08	10.637	0.0940	4.0064E+07	0.6646E-07
7	4.2508E+08	10.611	0.0942	4.0059E+07	0.4938E-07
8	4.2390E+08	10.589	0.0944	4.0029E+07	0.3237E-07
9	4.2277E+08	10.574	0.0945	3.9982E+07	0.2032E-07
10	4.2174E+08	10.561	0.0947	3.9934E+07	0.1465E-07
11	4.2072E+08	10.546	0.0948	3.9894E+07	0.1332E-07
12	4.1959E+08	10.525	0.0950	3.9867E+07	0.1365E-07
13	4.1819E+08	10.495	0.0953	3.9845E+07	0.1676E-07
14	4.1783E+08	10.471	0.0955	3.9903E+07	0.7113E-08
True values	4.1858E+08	10.465	0.0956	4.0000E+07.00	

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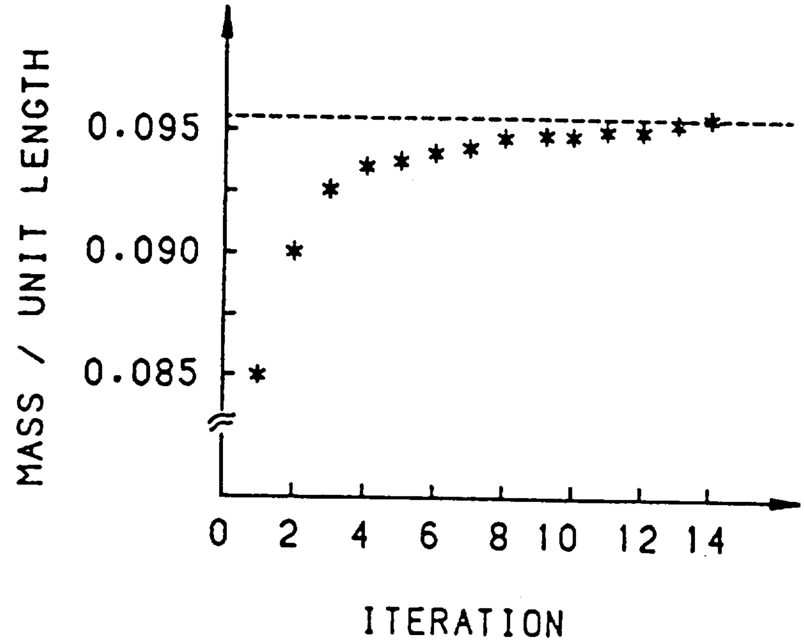
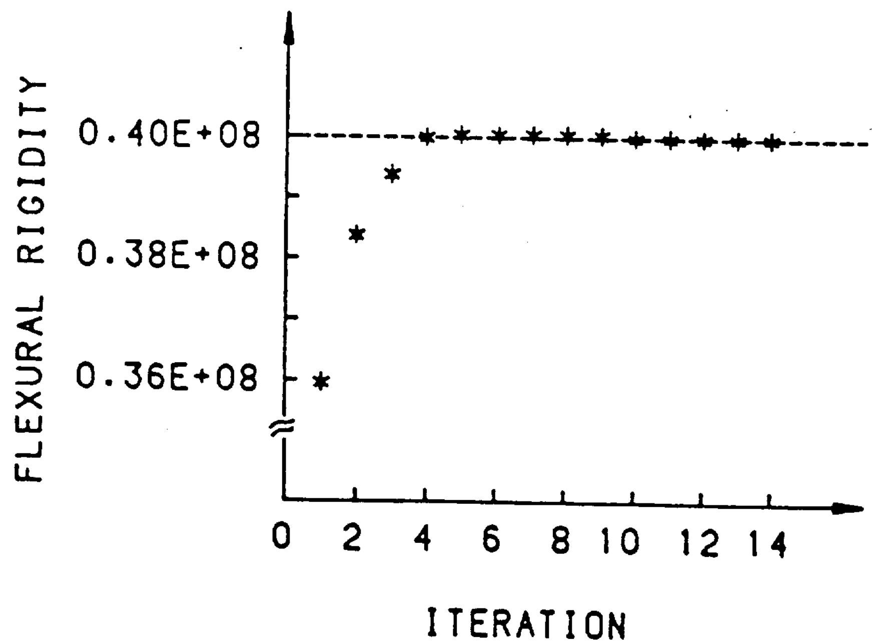
Table 3

Performance Data for Case I : Linearized SCOLE Model

Iteration	q_1	q_2	m	EI	$\int_0^T \text{error}^2\, dt$
1	4.2353E+08	11.765	0.0850	3.6000E+07	0.1341E-04
2	4.2531E+08	11.068	0.0904	3.8427E+07	0.1894E-05
3	4.2613E+08	10.809	0.0925	3.9422E+07	0.3051E-06
4	4.2744E+08	10.704	0.0934	3.9929E+07	0.9142E-07
5	4.2705E+08	10.665	0.0938	4.0041E+07	0.8040E-07
6	4.2618E+08	10.637	0.0940	4.0064E+07	0.6660E-07
7	4.2509E+08	10.611	0.0942	4.0060E+07	0.4955E-07
8	4.2392E+08	10.590	0.0944	4.0030E+07	0.3255E-07
9	4.2277E+08	10.574	0.0945	3.9982E+07	0.2032E-07
10	4.2175E+08	10.561	0.0947	3.9933E+07	0.1478E-07
11	4.2066E+08	10.546	0.0948	3.9888E+07	0.1378E-07
12	4.1957E+08	10.524	0.0950	3.9867E+07	0.1361E-07
13	4.1816E+08	10.495	0.0953	3.9846E+07	0.1667E-07
14	4.1773E+08	10.471	0.0955	3.9893E+07	0.8371E-08
True values	4.1858E+08	10.465	0.0956	4.0000E+07.00	



(a)



(b)

Fig. 5 Convergence of parameters for the SCOLE problem in Case I using nonlinear model.

(a) For ρA / unit length.

(b) For flexural rigidity, EI.

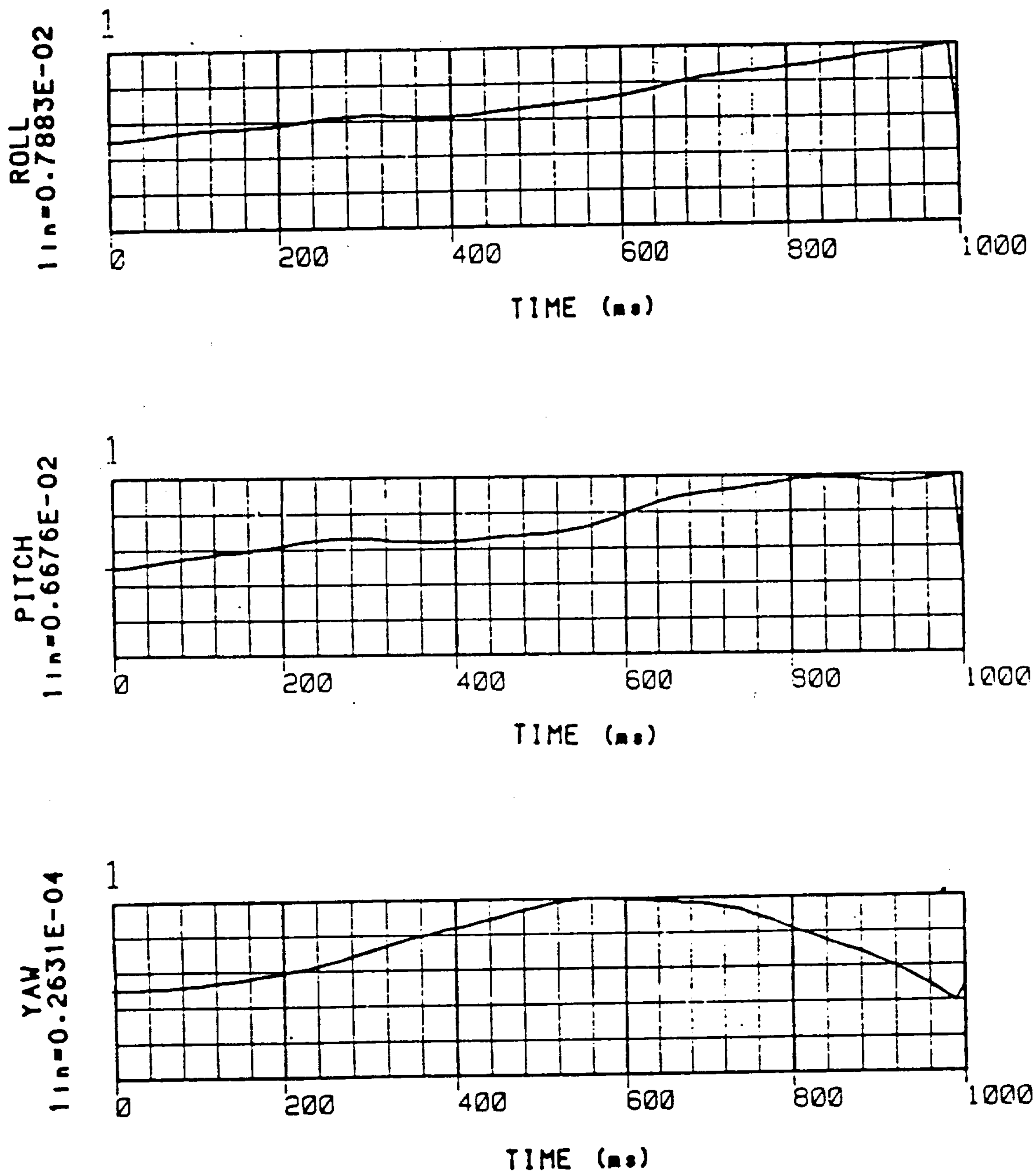


Fig. 4 Roll, pitch and yaw displacements when a damping factor of 0.003 is added to the system.

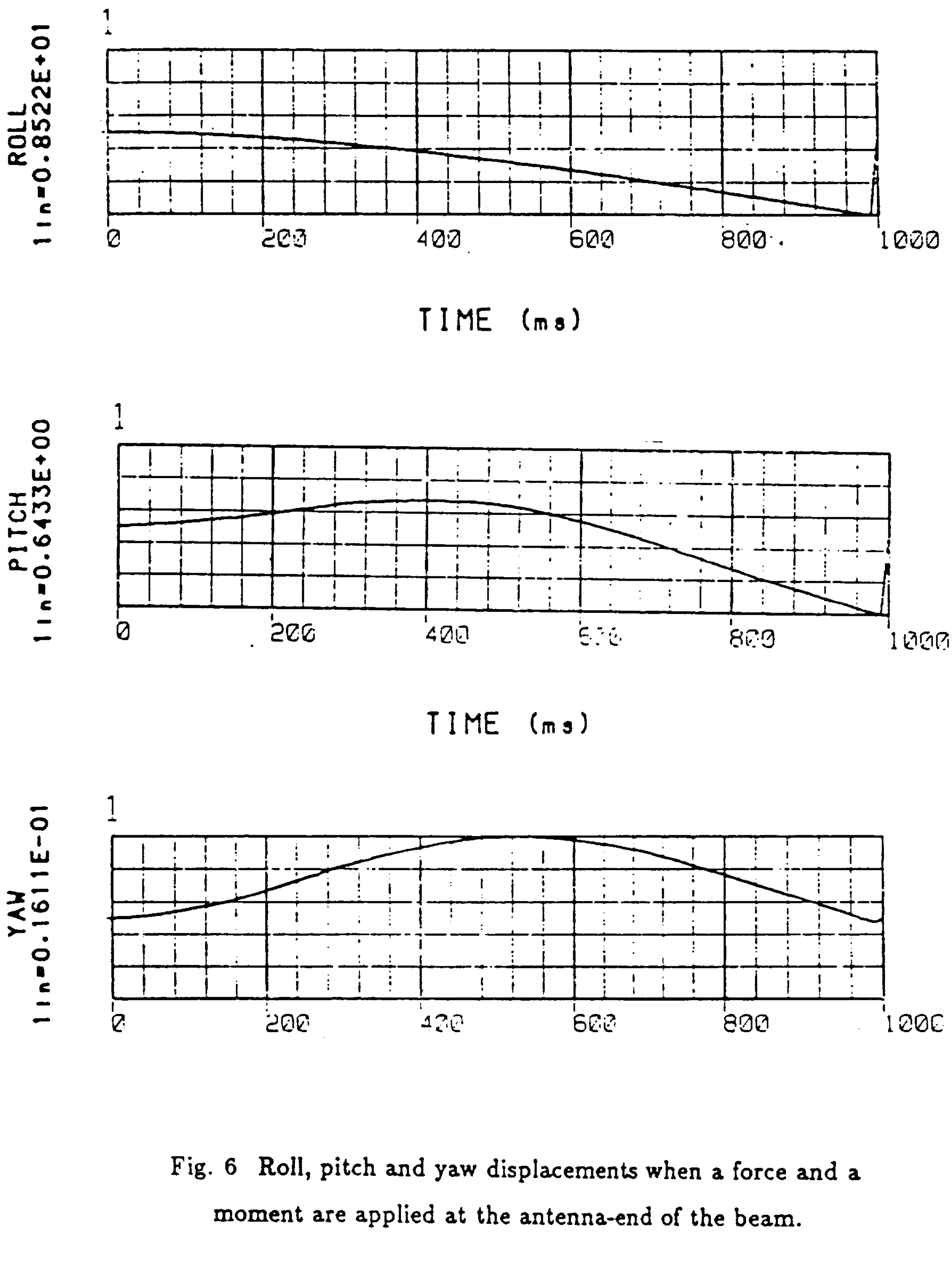
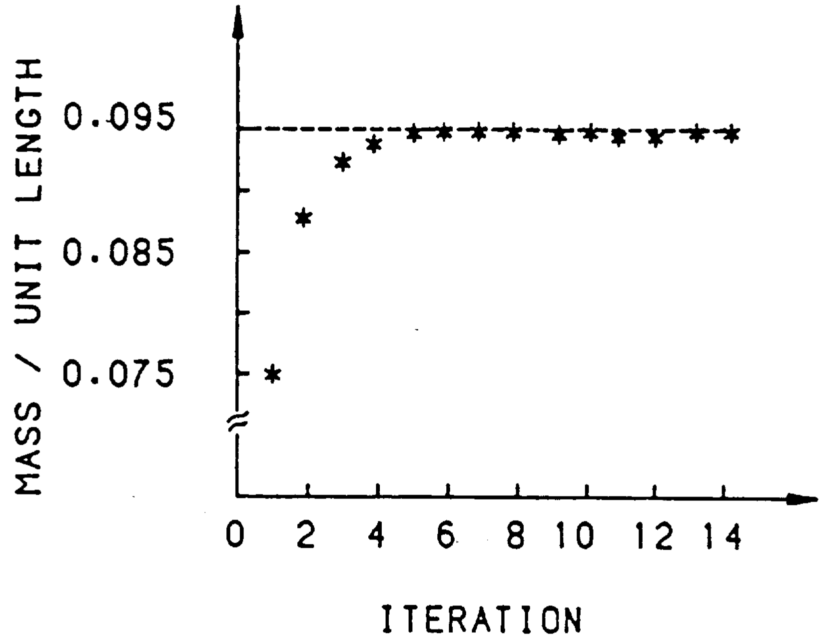


Fig. 6 Roll, pitch and yaw displacements when a force and a moment are applied at the antenna-end of the beam.

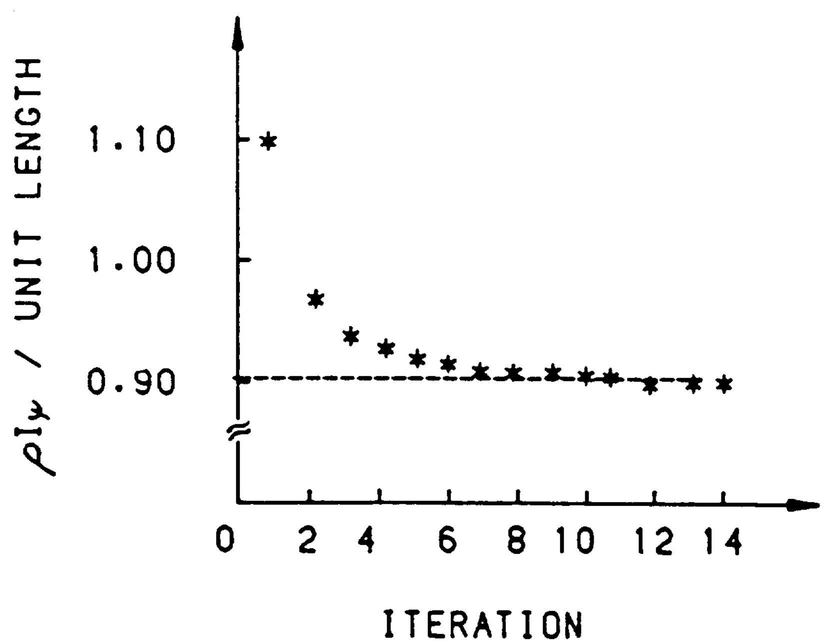
Table 4

Performance Data for Case II

Iteration	EI	ρA	ρI_ψ	ξ
1	32000000.00	0.0750	1.1000	0.00250
2	37582208.00	0.0881	0.9690	0.00250
3	39607652.00	0.0928	0.9428	0.00250
4	40157244.00	0.0941	0.9362	0.00250
5	40372424.00	0.0946	0.9247	0.00280
6	40338068.00	0.0945	0.9183	0.00288
7	40309128.00	0.0945	0.9127	0.00293
8	40282296.00	0.0943	0.9088	0.00289
9	40260868.00	0.0943	0.9047	0.00287
10	40243016.00	0.0942	0.9008	0.00285
11	40226244.00	0.0942	0.8983	0.00281
12	40202376.00	0.0942	0.8979	0.00286
13	40181576.00	0.0942	0.8977	0.00292
14	40162324.00	0.0941	0.8974	0.00299
15	40149520.00	0.0941	0.8973	0.00298
16	40140040.00	0.0941	0.8973	0.00295
True values	40000000.00	0.0956	0.9089	0.003



(a)



(b)

Fig. 7 Convergence of error criterion for Case II.

(a) For ρA / unit length.

(b) For ρI_ψ /unit length.

CONCLUSION

Infinite-dimensional identification method presented in this paper shows a significant promise in the parameter estimation of flexible structures with great potentials for applications to LSS's. The basic approach is the abstract formultion of the system dynamics in function spaces and then applying optimal control theory to adjust system parameters so that the error between actual and model data is minimized. The use of partial differential equation for the purpose of estimation elimainates many problems associated with model trunction in the finite dimensional approach. Based on partial differential equation models and a quadratic performance index an algorithm to estimate the optimal parameters has been developed. The numerical results show the effectiveness of the algorithm in estimating parameters of the flexible beam in the SCOLE problem. The results show fairly good match between the model and the estimated parameters. However, as the number of parameters to be identified increases it becomes increasingly time consuming and difficult to solve. Also, due to model mismatch, slightly less accuracies are expected if experimental measurement data from physical beam were used.

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